

THE PHYSICS SPACE DIMENSION

Gunn A. Quznetsov
quznets@geocities.com

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Abstract

All fermions and all interactions between fermions are expressed by the Cayley numbers in our space-time.

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1 3+1 SPACE-TIME

Let $A(t, \vec{x})$ be the event, which can be expressed as: "The particle e_A is detected in the space point \vec{x} at the time moment t " and $B(t_0, \vec{x}_0)$ - as:

"The particle e_B is detected in the space point \vec{x}_0 at the time moment t_0 ".

Let $\rho(t, \vec{x})$ be the probability density of the event A . That is

$$\int \int \int_{(V)} d\vec{x} \cdot \rho(t, \vec{x})$$

equals to the probability to detect the particle e_A in the space domain V at the time moment t .

Let $\rho_c(t, \vec{x}|t_0, \vec{x}_0)$ be the conditional probability density of the event A for the event B . That is

$$\int \int \int_{(V)} d\vec{x} \cdot \rho(t, \vec{x}|t_0, \vec{x}_0)$$

equals to the probability to detect the particle e_A in the space domain V at the time moment t , if the particle e_B is detected in the space point \vec{x}_0 at the time moment t_0 .

In this case, if

$$\rho_c(t, \vec{x}|t_0, \vec{x}_0) = g(t, \vec{x}|t_0, \vec{x}_0) \cdot \rho(t, \vec{x}),$$

then the function $g(t, \vec{x}|t_0, \vec{x}_0)$ is the interaction function for e_A and e_B . If $g(t, \vec{x}|t_0, \vec{x}_0) = 1$ then the particles e_A and e_B do not interact.

In the Quantum Theory a probability density equals to the quadrate of the state vector module. A fermion state vector is the 4-component complex vector. Hence, a fermion state vector has got 8 real components. Therefore, some conformity between such vectors and the octaves (the Cayley numbers) can be determined.

Let $\rho = \Psi^\dagger \cdot \Psi$ and $\rho_c = \Psi_c^\dagger \cdot \Psi_c$. Here: Ψ and Ψ_c are the 4-component complex state vectors. And Ψ and Ψ_c are the octaves by this conformity.

Because the Cayley algebra is the division algebra, then the octave φ exists, for which: $\Psi_c = \varphi \bullet \Psi$ (here \bullet is the symbol of the algebra Cayley product).

Because the Cayley algebra is the normalized algebra, then $g = \varphi^\dagger \cdot \varphi$.

Therefore, all fermion interactions can be expressed by the octaves in the $3 + 1$ space-time.

2 $\mu+1$ SPACE-TIME

Let us consider the probability density $\rho(t, \vec{x})$ [1] for some point-event $A(t, \vec{x})$ in the $\mu + 1$ space-time. That is

$$\int_{(V)} d\vec{x} \cdot \rho(t, \vec{x})$$

is the probability for A to happen in the space domain (V) at the time moment t in the $\mu + 1$ space-time.

Let $\vec{j}(t, \vec{x})$ be the probability current vector [2]. In this case

$$\langle \rho(t, \vec{x}), \vec{j}(t, \vec{x}) \rangle$$

is the probability density $\mu + 1$ vector.

The Clifford set of the range s is the set K of the $s \times s$ complex matrices for which:

- 1) if $\gamma \in K$ then $\gamma^2 = 1$ (here 1 is the identity $s \times s$ matrix);
- 2) if $\gamma \in K$ and $\beta \in K$ then $\gamma \cdot \beta + \beta \cdot \gamma = 0$ (here 0 is the zero $s \times s$ matrix);
- 3) There does not exist the $s \times s$ matrix ζ which anticommutes with all K elements, for which $\zeta^2 = 1$, and which is not the element of K .

For example, the Clifford pentad $\langle \beta^1, \beta^2, \beta^3, \beta^4, \gamma^0 \rangle$ [3] is the Clifford set of the range 4.

By [4] for every natural number z the Clifford set of the range 2^z exists.

For every probability density vector $\langle \rho(t, \vec{x}), \vec{j}(t, \vec{x}) \rangle$ the natural number s , the Clifford set K of range s and the complex s -vector $\Psi(t, \vec{x})$ exist, for which: $\gamma_n \in K$ and

$$\Psi(t, \vec{x})^\dagger \cdot \Psi(t, \vec{x}) = \rho(t, \vec{x}), \quad (1)$$

$$\Psi(t, \vec{x})^\dagger \cdot \gamma_n \cdot \Psi(t, \vec{x}) = j_n(t, \vec{x}). \quad (2)$$

In this case let $\Psi(t, \vec{x})$ be named as the s -spinor for $\langle \rho(t, \vec{x}), \vec{j}(t, \vec{x}) \rangle$.

If $\langle \rho(t, \vec{x}), \vec{j}(t, \vec{x}) \rangle$ obeys to the continuity equation [5] then from (1), (2): $\Psi(t, \vec{x})$ is fulfilled to the Dirac equation generalization on the $\mu + 1$ space-time.

Let $\rho_c(t, \vec{x}|t_0, \vec{x}_0)$ be the conditional probability density of the event A for the event B , but in the $\mu + 1$ space-time, too. And if

$$\rho_c(t, \vec{x}|t_0, \vec{x}_0) = g(t, \vec{x}|t_0, \vec{x}_0) \cdot \rho(t, \vec{x}),$$

then the function $g(t, \vec{x}|t_0, \vec{x}_0)$ is the interaction function for e_A and e_B , too, but in the $\mu + 1$ space-time.

Let Ψ_c and Ψ be the s -spinors, for which: $\rho = \Psi^\dagger \cdot \Psi$ and $\rho_c = \Psi_c^\dagger \cdot \Psi_c$.

Let Ψ_c and Ψ are the elements of the algebra \mathfrak{S} with the product $*$, and for every Ψ_c and Ψ the element φ of \mathfrak{S} exists, for which: $\Psi_c = \varphi * \Psi$ and $\varphi^\dagger \cdot \varphi = g$.

In this case \mathfrak{S} is the division normalized algebra and the \mathfrak{S} dimension is not more than 8 from the Hurwitz theorem [6] (The every normalized algebra with the unit is isomorphic to alone from the followings: the real numbers algebra R , the quaternions algebra K , or the octaves algebra \mathcal{O}) and from the generalized Frobenius theorem [7] (The division algebras have got the dimension 1,2,4 or 8, only). Therefore, the Clifford set matrices size are not more than 4×4 (the Clifford matrices are the complex matrices). Such Clifford set contains not more than 5 elements. The diagonal elements of this pentad defines the space, in which the physics particle move. This space dimension is not more than 3 [8]. Hence, in this case we have got 3+1 space.

If $\mu > 3$ then for every algebra the interaction functions exist, which do not belong to this algebra. I'm name such interactions as the supernatural for this algebra interactions.

3 RESUME

1. All fermions and all interactions between fermions are expressed by the octaves in our space-time.
2. The probability, which is defined by the relativistic $\mu + 1$ -vector of the probability density, fulfils to the Quantum Theory principles.
3. In the $\mu+1$ space-time: if $\mu \leq 3$ then the supernatural interactions do not happen, if $\mu > 3$ then the supernatural interactions happen.

4 APPENDIX. CAYLEY ALGEBRA

The Cayley algebra \mathcal{O} has got basis on the 8 dimensional real linear space. The orthogonal normalized basic elements of \mathcal{O} are: $1, i, j, k, E, I, J, K$. The product of \mathcal{O} is defined by the following rules ($\hat{a} \bullet \hat{e}$):

$\hat{a} \backslash \hat{e}$	1	i	j	k	E	I	J	K
1	1	i	j	k	E	I	J	K
i	i	-1	k	$-j$	I	$-E$	$-K$	J
j	j	$-k$	-1	i	J	K	$-E$	$-I$
k	k	j	$-i$	-1	K	$-J$	I	$-E$
E	E	$-I$	$-J$	$-K$	-1	i	j	k
I	I	E	$-K$	J	$-i$	-1	$-k$	j
J	J	K	E	$-I$	$-j$	k	-1	$-i$
K	K	$-J$	I	E	$-k$	$-j$	i	-1

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